$$1496=15^2+1271;\ 1271=41\times 31=36^2-5^2.$$
$$\therefore\ S-15^2=36^2-5^2, \text{ or } S-15^2+5^2=36^2,$$
$$\text{or } 1^2+2^2+\ldots.14^2+16^2+5^2=36^2.$$

Corollary.—If the square number $(2n^2_1-1)^2$ taken from S is one of the square numbers in S, as in the above example and which may always be the case, the formula still furnishes the means of finding n square numbers whose sum is a square number.

We may write at once, $(n+1)^2-1^2=n(n+2)$

$$(n+2)^2-2^2=n(n+4)$$
$$(n+m)^2-m^2=n(n+2m).$$

Hence we see the form that two factors must have, in order that their product may be equal to the difference between two squares.

We see that the factors are both odd or both even.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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CHAPTER THIRD.

On the Continuity of Space.

[Continued from the July Number].

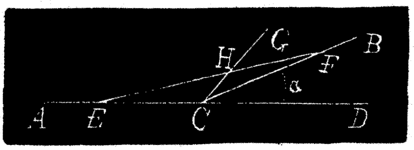
It is here thought best to interpolate some expository matter regarding parts of elementary geometry which involve the difficult idea of *continuity.*

When a mathematician says: "There may be a triangle whose angle-sum differes from a straight angle by less than any *given* finite angle however small," the meaning is simply, "give me geometrically any one particular finite angle you choose, and I will prove geometrically that a triangle may exist the sum of whose three interior angles differs from two right angles by less than that particular angle you have given me."

The problem: "To construct a triangle whose angle-sum differs from a straight angle by less than any given finite angle however small," means, "if any one particular finite angle is given graphically, show how geometrically to construct a triangle whose angle-sum differs from two right angles by less than that one particular *given* finite angle."

The same easy solution of this problem is true both in Euclid and in Lobatschewsky.

Solution: Let *GCD* be the given finite angle however small. Extend the ray *CD* through *C* to *A*. In the ray *CA* take any point *E*. Join *E* to any point of the ray *CG*, as *H*. The constructed triangle *ECH* has an angle sum differing from a straight angle by less than *GCD*.



Proof: The angle *ECH* differs from a straight angle by the angle *GCD*. Therefore the sum [not greater than a straight angle] of *ECH* and *CEH* and *CHE* differs from a straight angle by less than the given angle *GCD*.

To see that the solution is not restricted by the smallness of the *given* finite angle, notice that if the smaller angle *DCB* were given we need only join *E* to any point *F* in the ray *CB* to get a triangle *ECF* which is a solution for the given angle *DCB*.

Thus we have proved that "There may be a triangle whose angle sum differs from a straight angle by less than any given finite angle however small," in the best possible way, namely by showing how to construct it geometrically.

Note that in general a proof that there may be a specified geometric entity, does not necessarily carry with it the possibility of its geometric construction. For example, there may be an angle which is one third of any given finite angle however small, yet is the possibility of its construction so far from being granted, that in fact the trisection of an angle is one of the three great insoluble geometric problems, ranking with the quadrature of the circle and the duplscation of the cube.

But we have just proved that in Euclid as in Lobatschewsky we may actually construct "a triangle whose angle sum is equal to two right angles minus the angle a which is less than any particular given finite angle b however small." In Euclid $a=0$. In Lobatschewsky a, though greater than zero is less than the particular given finite angle b.

But suppose there were any need for introducing infinitesimal angles, would we be justified in saying "the difference between a finite angle less than two right angles and two right angles is necessarily finite."

Assuredly not. For a straight angle minus an infinitesimal would be finite; but the difference between this finite angle and two right angles would be be only that infinitesinal.